NON-PERTURBATIVE RENORMALIZATION GROUP FUNCTIONS IN (2 + 1) - DIMENSIONAL SUPERSYMMETRIC GAUGE THEORIES

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ABSTRACT. Three-dimensional supersymmetric Higgs models with an additional U(N) 'flavor' symmetry are considered within the $1/N$ expansion. Explicit expressions for the renormalization group functions are obtained in the large N limit which exhibit logarithmic dependence on the gauge coupling constants.

1. In the last few years the $1/N$ expansion in quantum field theory, N being the number of 'flavor' or 'color' degrees of freedom respectively, became a powerful method for attacking non-perturbative problems (phase transitions, confinement etc.) in gauge theories (for a review and recent developments, see [1]).

Since the renormalization group (RG) is of major interest, it is important to invoke the $1/N$ expansion for obtaining essentially non-perturbative RG functions which, in particular, could yield non-trivial infrared (IR) and ultraviolet (UV) stable fixed points governing the scaling behavior of the theory at low and high energies respectively. However, it is extremely difficult to realize this task consistently in four-dimensional gauge field models because of the lack of a systematic $1/N$ expansion for them so far [1].

In the present note we consider a simpler model – the supersymmetric (SS) Higgs model (with $U(N)_{\text{flavor}} \times U(n)_{\text{color gauge}}$ internal symmetry) in $D = 2 + 1$ space-time dimensions (HM_3). Being relatively easily tractable, it displays at the same time a rich enough structure within the $1/N$ expansion (see Section 3 below) analogous to one of the $D = 2 + 1$ SS generalized non-linear sigma models (GNLSMs) [2]. The main results are the explicit expressions for the RG functions (Gell-Mann–Low beta functions and the anomalous dimensions of the superfields) in the large N limit, whose non-perturbative character is exhibited through the logarithmic dependence on the gauge coupling constants (Equations (11) and (12) below).★

2. The superspace [5] Lagrangian of SS HM_3 reads:

$$\mathcal{L}(x, \theta) = \tfrac{1}{2}(\bar{\nabla}_\alpha \Phi^*)^k_a (\nabla_\alpha \Phi)^k_a - \frac{g_0}{2Nn}\left(\Phi^{*k}_a \Phi^k_a - \frac{Nn\mu}{T}\right)^2 - \frac{g_1}{2Nn}(\Phi^* \lambda_A \Phi)^2 +$$

★Logarithmic dependence on gauge coupling constants has already been found in the usual non-SS massless spinor QED_3 and QCD_3 [3] as well as in the $1/N$ expanded non-SS scalar QED_3 [4].

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$$+\frac{iNn}{8e_0^2\mu}\operatorname{tr}(\hat{\mathscr{F}}_0\hat{\partial}\hat{\mathscr{F}}_0)+\frac{iN}{8e_1^2\mu}\operatorname{tr}(\hat{\underset{\sim}{\mathscr{F}}}\hat{\partial}\hat{\underset{\sim}{\mathscr{F}}}+\tfrac{2}{3}\hat{\underset{\sim}{\mathscr{F}}}\hat{\underset{\sim}{\mathscr{F}}}\hat{\underset{\sim}{\mathscr{F}}})-nN\mathscr{B}_0\overline{\mathscr{D}}_\alpha\mathscr{A}_{0\alpha}-$$

$$-N\operatorname{tr}(\underset{\sim}{\mathscr{B}}\overline{\mathscr{D}}_\alpha\underset{\sim}{\mathscr{A}}_\alpha)+\operatorname{tr}(\underset{\sim}{\chi}^*\overline{\mathscr{D}}_\alpha\nabla_\alpha\underset{\sim}{\chi}), \tag{1}$$

with the following notations

$$\hat{\mathscr{F}}_{0\alpha\beta}=\tfrac{1}{2}(\mathscr{D}_\alpha\overline{\mathscr{A}}_{0\beta}+\overline{\mathscr{D}}_\beta\mathscr{A}_{0\alpha}),\qquad \hat{\underset{\sim}{\mathscr{F}}}_{\alpha\beta}=\tfrac{1}{2}(\mathscr{D}_\alpha\overline{\underset{\sim}{\mathscr{A}}}_\beta+\overline{\mathscr{D}}_\beta\underset{\sim}{\mathscr{A}}_\alpha+i\{\underset{\sim}{\mathscr{A}}_\alpha,\overline{\underset{\sim}{\mathscr{A}}}_\beta\}),$$

$$(\nabla_\alpha\Phi)_a^k=\mathscr{D}_\alpha\Phi_a^k+i\mathscr{A}_{0\alpha}\Phi_a^k+i\underset{\sim}{\mathscr{A}}_\alpha^{kl}\Phi_a^l,\qquad (\nabla_\alpha\underset{\sim}{\chi})^{kl}=\mathscr{D}_\alpha\underset{\sim}{\chi}^{kl}+i[\underset{\sim}{\mathscr{A}}_\alpha,\underset{\sim}{\chi}]^{kl},$$

$$\mathscr{D}_\alpha=\partial/\partial\overline{\theta}^\alpha-i(\hat{\partial}_x\theta)_\alpha,\qquad \hat{X}\equiv X^\mu\gamma_\mu,\qquad \overline{X}_\alpha\equiv\mathscr{C}_{\alpha\beta}^{-1}X_\beta,\qquad \mathscr{C}^{-1}\gamma_\mu\mathscr{C}=-\gamma_\mu^T,$$

$$\underset{\sim}{X}^{kl}\equiv X_A\lambda_A^{kl},\qquad \operatorname{tr}(\lambda_A\lambda_B)=n\delta_{AB},$$

where μ is an arbitrary mass scale, and T, $g_{0,1}$, $e_{0,1}$ are dimensionless coupling constants. The auxiliary superfields $\mathscr{B}_0$, $\underset{\sim}{\mathscr{B}}$ enforce Landau gauge conditions for the Majorana fermionic Abelian and non-Abelian gauge superfields $\mathscr{A}_0$, $\underset{\sim}{\mathscr{A}}$*; $\underset{\sim}{\chi}$ are the corresponding Faddeev–Popov superfield ghosts. The Hermitian $n\times n$ matrices λ_A, $A=1,\dots,n^2-1$, spans a hermitian basis of the SU(n) Lie algebra. The summation over repeated indices ('flavor' ones a, $b=1,\dots,N$; 'color' ones k, $l=1,\dots,n$; adjoint-SU(n) ones A, $B=1,\dots,n^2-1$ and Lorentz-spinor ones $\alpha,\beta=1,2$) is understood and the latter will often be suppressed for brevity. In component fields the model (1) looks as:

$$\mathcal{L}(x)=(\nabla_\mu\varphi)^*(\nabla^\mu\varphi)+\frac{i}{2}\overline{\psi}\,\overset{\leftrightarrow}{\hat{\nabla}}\,\psi-\frac{Nn}{4e_0^2\mu}F_{\kappa\lambda}^2(A_0)-\frac{N}{4e_1^2\mu}\operatorname{tr}(\underset{\sim}{F}_{\kappa\lambda}^2(\underset{\sim}{A}))-$$

$$-NnB_0\partial_\mu A_0^\mu-N\operatorname{tr}(\underset{\sim}{B}\partial_\mu\underset{\sim}{A}^\mu)+\operatorname{tr}(\underset{\sim}{\eta}^*\partial^\mu\nabla_\mu\underset{\sim}{\eta})+\frac{Nn}{8e_0^2\mu}\overline{C}_0 i\hat{\partial}C_0+$$

$$+\frac{Ni}{8e_1^2\mu}\operatorname{tr}(\underset{\sim}{\overline{C}}\hat{\nabla}\underset{\sim}{C})+\frac{i}{2}\overline{\psi}(C_0+\underset{\sim}{C})\varphi^*-\frac{i}{2}\varphi^*(\overline{C}_0+\underset{\sim}{\overline{C}})\psi-$$

$$-g_0(Nn)^{-1}[(\overline{\psi}\varphi)(\varphi^*\psi)-\tfrac{1}{2}(\overline{\psi}\varphi)\mathscr{C}^{-1}(\overline{\psi}\varphi)-\tfrac{1}{2}(\varphi^*\psi)\mathscr{C}^{-1}(\varphi^*\psi)]- \tag{1'}$$

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*The kinetic term for $\underset{\sim}{\mathscr{A}}(x,\theta)$:

$$i(8e_1^2\mu)^{-1}\operatorname{tr}(\hat{\underset{\sim}{\mathscr{F}}}\hat{\partial}\hat{\underset{\sim}{\mathscr{F}}}+\tfrac{2}{3}\hat{\underset{\sim}{\mathscr{F}}}\hat{\underset{\sim}{\mathscr{F}}}\hat{\underset{\sim}{\mathscr{F}}})=(4e_1^2\mu)^{-1}\epsilon^{\lambda\mu\nu}\operatorname{tr}(\underset{\sim}{\mathscr{F}}_\lambda\partial_\mu\underset{\sim}{\mathscr{F}}_\nu+\tfrac{2}{3}\underset{\sim}{\mathscr{F}}_\lambda\underset{\sim}{\mathscr{F}}_\mu\underset{\sim}{\mathscr{F}}_\nu)$$

closely resembles the instanton density in the usual (non-SS) $D=4$ Yang–Mills theories. This is due to the fact that under superfield gauge transformations ($\Omega(x,\theta)\in SU(n)$):

$$\underset{\sim}{\mathscr{A}}'_\alpha=\Omega^{-1}(\underset{\sim}{\mathscr{A}}_\alpha+\mathscr{D}_\alpha)\Omega,\qquad \hat{\underset{\sim}{\mathscr{F}}}'_{\alpha\beta}=\Omega^{-1}(\hat{\mathscr{F}}_{\alpha\beta}+\hat{\partial}_{\alpha\beta})\Omega.$$

$$- g_1(Nn)^{-1}[(\bar\psi\lambda_A\varphi)(\varphi^*\lambda_A\psi) - \tfrac{1}{2}(\bar\psi\lambda_A\varphi)\mathscr{C}^{-1}(\bar\psi\lambda_A\varphi) - \tfrac{1}{2}(\varphi^*\lambda_A\psi)\mathscr{C}^{-1}(\varphi^*\lambda_A\psi)] +$$

$$+ g_0(Nn)^{-1}\bar\psi\psi(\varphi^*\varphi - Nn\mu/T) + g_1(Nn)^{-1}(\bar\psi\lambda_A\psi)(\varphi^*\lambda_A\varphi) -$$

$$- \mu(NnT)^{-1}[g_0^2(\varphi^*\varphi - Nn\mu/T)^2 + g_1^2(\varphi^*\lambda_A\varphi)^2] - g_0^2(Nn)^{-2}(\varphi^*\varphi - Nn\mu/T)^3 -$$

$$- g_1(Nn)^{-2}d_{ABC}(\varphi^*\lambda_A\varphi)(\varphi^*\lambda_B\varphi)(\varphi^*\lambda_C\varphi);$$

$$F_{\kappa\lambda}(A_0) = \partial_\kappa A_{0\lambda} - \partial_\lambda A_{0\kappa}; \quad \underset{\sim}{F}_{\kappa\lambda}(\underset{\sim}{A}) = \partial_\kappa \underset{\sim}{A}_\lambda - \partial_\lambda \underset{\sim}{A}_\kappa + i[\underset{\sim}{A}_\kappa, \underset{\sim}{A}_\lambda],$$

$$\{\lambda_A, \lambda_B\} = 2(\delta_{AB} + d_{ABC}\lambda_C), \quad (\nabla_\mu\varphi)_a^k = \partial_\mu\varphi_a^k + iA_{0\mu}\varphi_a^k + i\underset{\sim}{A}_\mu^{kl}\varphi_a^l \text{ etc.},$$

where B_0, $\underset{\sim}{B}$ and $\underset{\sim}{\eta}$ are the usual Landau and Faddeev–Popov ghost fields and C_0, $\underset{\sim}{C}$ are Majorana spinors transforming under the singlet and the adjoint representation of SU(n) respectively.

3. The $1/N$ expansion of SS HM_3 is generated within the functional integral formalism exactly along the same lines as in the case of SS GNLSMs [2], starting from an equivalent form of (1) with the auxiliary superfields Σ_0, $\underset{\sim}{\Sigma}$:

$$\mathscr{L}'(x,\theta) = \tfrac{1}{2}(\bar\nabla_\alpha\Phi^*)(\nabla_\alpha\Phi) - \Phi^*(\Sigma_0 + \underset{\sim}{\Sigma})\Phi + Nn\mu/T\,\Sigma_0 + Nn(2g_0)^{-1}\Sigma_0^2 +$$

$$+ N(2g_1)^{-1}\,\mathrm{tr}\,(\underset{\sim}{\Sigma}^2) + iNn(8e_0^2\mu)^{-1}\,\mathrm{tr}\,(\hat{\mathscr{F}}_0\hat\partial\hat{\mathscr{F}}_0) - Nn\mathscr{B}_0\bar{\mathscr{D}}_\alpha\mathscr{A}_{0\alpha} + \qquad (2)$$

$$+ iN(8e_1^2\mu)^{-1}\,\mathrm{tr}\,(\hat{\underset{\sim}{\mathscr{F}}}\hat\partial\hat{\underset{\sim}{\mathscr{F}}}) + \tfrac{2}{3}\hat{\underset{\sim}{\mathscr{F}}}\hat{\underset{\sim}{\mathscr{F}}}\hat{\underset{\sim}{\mathscr{F}}}) - N\,\mathrm{tr}\,(\underset{\sim}{\mathscr{B}}\bar{\mathscr{D}}_\alpha\underset{\sim}{\mathscr{A}}_\alpha) + \mathrm{tr}\,(\underset{\sim}{\chi}^*\bar{\mathscr{D}}_\alpha\nabla_\alpha\underset{\sim}{\chi}).$$

Therefore, we shall only list the resulting features.

SS HM_3 undergo a second-order phase transition at a certain critical value T_c of T (the 'temperature'). The corresponding high-temperature phase (HTP) is SS, $U(N) \times U(n)_{\mathrm{gauge}}$ symmetric with a particle spectrum consisting of $n \cdot N$ boson-fermion pairs (φ- and ψ-quanta in (1′)) of equal dynamically-generated mass

$$m = 4\pi\mu(1/T_c - 1/T)(1 - 4\pi/g_0)^{-1}, \quad T > T_c, \qquad (3)$$

and of a massless transverse photon and $n^2 - 1$ gluons ($A_{0\mu}$- and $\underset{\sim}{A}_\mu$-quanta in (1′)). The Higgs–Goldstone low-temperature phase (LTP) is SS with spontaneous breaking of the internal $U(N) \times U(n)_{\mathrm{gauge}}$ symmetry due to the non-zero vacuum expectation value

$$\langle\Phi_a^k(x,\theta)\rangle = N^{1/2}V_a^k + 0(N^{-1})); \quad V_a^k = 0,\ a = n+1, \ldots, N; \ v_a^{*k}v_a^l \equiv \delta^{kl}|v|^2$$
$$= \mu(1/T - 1/T_c)\delta^{kl}, \quad T < T_c, \qquad (4)$$

up to the residual $U(N-n) \times \mathrm{diag}(U(n)_{\mathrm{flavor}} \times U(n)_{\mathrm{gauge}})$ symmetry (the stability subgroup of

LTP vacuum (4)) and its particle spectrum consists of only $n \cdot (N-1)$ pairs of massless Goldstone bosons and fermions (φ_a^k, ψ_a^k, $a = n+1, \ldots, N$ and $v_a^{*l}\varphi_a^k$, $v_a^{*l}\psi_a^k$, $l \neq k$); the remaining $v_a^{*k}\varphi_a^k$, $v_a^{*k}\psi_a^k$-fields (no summation over k) as well as the would-be (via the Higgs mechanism) massive photon and gluons are here 'confined'. Also the Majorana fields C_0, $\underset{\sim}{C}$, in spite of the presence of the corresponding kinetic terms for them in (1′), completely disappear from the spectrum★.

The above information is essentially contained in the 'free' propagators of the corresponding $1/N$ supergraph technique. We write down the latter in a unified notation simultaneously suited for both HTP and LTP, as well as for the pre-scaling $T = T_c$ theory, and we introduce an auxiliary parameter s ($0 \leqslant s \leqslant 1$) in view of the application of a SS version [6] of the 'soft mass' re-normalization scheme [7] later on (Section 4) (the propagators themselves are obtained by setting $s = 1$ and $v = 0$ (HTP), $m = 0$ (LTP) or $v = 0$, $m = 0$ ($T = T_c$)):

$$
\begin{aligned}
\langle \Phi_a^k \Phi_b^{*l} \rangle^{(0)} &= (-i)\delta_{ab}\,\delta^{kl}\,\mathrm{e}^{\bar{\theta}\hat{p}\varphi}[(sm + (1-s)\mu)^2 - p^2]^{-1} + \\
&\quad + in^{-1}[(1-s)^2\mu^2 - p^2]^{-2}\,\mathrm{e}^{\bar{\theta}\hat{p}\varphi}\{v_a^k v_b^{*l} H(g_0) + (\lambda_A v_a)^k (v_b^* \lambda_A)^l H(g_1)\}, \\
\langle \Sigma_0 \Sigma_0 \rangle^{(0)}(g_0) &= (iNn)^{-1}\{\mathrm{e}^{\bar{\theta}\hat{p}\varphi}(F - s|v|^2/p^2) - \\
&\quad - \delta(\theta - \varphi)(2smF + 1/g_0)\}[-p^2(F - s|v|^2/p^2)^2 + (2smF + 1/g_0)^2]^{-1}, \\
\langle \mathscr{A}_{0\alpha} \bar{\mathscr{A}}_{0\beta} \rangle^{(0)}(e_0) &= i(Nn)^{-1}[\tfrac{1}{2}\hat{p}(2sm - \hat{p})F + s|v|^2 - p^2(2e_0^2\mu)^{-1}]^{-1}_{\alpha\gamma}\Pi_{\gamma\beta}, \\
\langle \Sigma_A \Sigma_B \rangle^{(0)}(g_1) &= \delta_{AB}\langle \Sigma_0 \Sigma_0 \rangle(g_1), \qquad \langle \mathscr{A}_{A\alpha} \bar{\mathscr{A}}_{B\beta} \rangle^{(0)}(e_1) = \delta_{AB}\langle \mathscr{A}_{0\alpha} \bar{\mathscr{A}}_{0\beta} \rangle(e_1),
\end{aligned}
\tag{5}
$$

where

$$
\begin{aligned}
H(g) &\equiv H(p^2, \theta - \varphi; g, |v|) = 8[(-p^2)^{1/2} + 8s|v|^2 + 8p^2/g\,\delta(\theta - \varphi)] \times \\
&\quad \times [(1 + 8s|v|^2(-p^2)^{-1/2})^2 + (8/g)^2]^{-1}, \\
F &\equiv F(p^2; sm) = (8\pi)^{-1}\int_0^1 \mathrm{d}x\,[(sm)^2 - p^2 x(1-x)]^{-1/2}, \\
\Pi_{\alpha\beta} &\equiv \Pi_{\alpha\beta}(p; \theta, \varphi) = \tfrac{1}{2}\mathrm{e}^{\bar{\theta}\hat{p}\varphi}[\delta_{\alpha\beta}\,\delta(\theta - \varphi) - (\hat{p}^{-1})_{\alpha\beta}].
\end{aligned}
\tag{6}
$$

Let us note that on classical level:

$$
\lim_{g_{0,1},\, e_{0,1} \to \infty} \mathcal{L}'(\mathrm{SSHM}_3, (2)) = \mathcal{L}(\mathrm{SS\ GNLSM}), \tag{7}
$$

see [2, 6].

★ $\langle C_0 \bar{C}_0 \rangle^{(0)}(e_0) = 2i(Nn)^{-1}[mF + \frac{1}{2}\hat{p}(F - |v|^2/p^2 + (e_0^2\mu)^{-1})]^{-1}$, $\langle C_A \bar{C}_B \rangle^{(0)}(e_1) = \delta_{AB}\langle C_0 \bar{C}_0 \rangle^{(0)}(e_1)$, $F \equiv F(p^2; m)$ is defined in (6).

According to Equation (13) below, (7) has a non-trivial meaning on quantum level as IR scaling limit of SS HM_3.

4. To formulate a unified renormalization procedure for the $1/N$ expansion of (2), it is most appropriate to use the same 'soft mass' renormalization scheme as for SS GNLSMs [6]. It is based on Bogoliubov's R-operation recurrence formula [8] (see Equations (4) in [6]). The corresponding subtraction operators $\tau^{\delta(\Gamma)}$, Γ being a one-particle irreducible supergraph, are defined by the properties (cf. [9]):

$$\tau^{\delta} f(p, s) = t^{\delta}_{p,s} f(p, s), \qquad \tau^{\delta} \theta_{\alpha} f(p, s; \theta) = \theta_{\alpha} \tau^{\delta + 1/2} f(p, s; \theta), \tag{8}$$

where $f(p, s)$, $f(p, s; \theta)$ are arbitrary functions and $t^{\delta}_{p,s}$ is the standard Taylor subtraction operator of order δ in the variables p (external momenta of Γ) and s around $p = 0, s = 0$. After implementing all subtractions, one sets $s = 1$. UV degrees (dimensions) $\delta(\Gamma)$ in (2) are taken to be equal to the corresponding canonical ones in the associated GNLSM (7):

$$\delta(\Gamma) = 3 - \tfrac{1}{2}(L_{\Phi}(\Gamma) + L_{\mathscr{A}}(\Gamma)) - L_{\Sigma}(\Gamma) - V_e(\Gamma),$$

where $L_{\Phi, \mathscr{A}, \Sigma}(\Gamma)$ and $V_e(\Gamma)$ are the numbers of external Φ-, $\mathscr{A}$-, Σ-lines and external vertices of Γ, respectively. On the other hand, due to the kinetic terms for $\mathscr{A}_0$ and $\underset{\sim}{\mathscr{A}}$ in (2), the canonical UV degrees $d(\Gamma)$ according to (5) are:

$$d(\Gamma) = \delta(\Gamma) - \mathfrak{L}_{\mathscr{A}}(\Gamma),$$

where $\mathfrak{L}_{\mathscr{A}}(\Gamma)$ denotes the number of internal $\mathscr{A}$-lines. Thus a non-canonical (oversubtraction-) renormalization scheme for SS HM_3 is employed (cf., [10] for the analogous non-SS models) which is needed to get (graph by graph within the renormalized $1/N$ expansion) a continuous limit (7).

As in [6] one can easily verify the following properties:

(i) UV divergences of (2) are at most logarithmic.

(ii) No artificial IR divergences are created through UV subtractions in the massless LTP and $T = T_c$ theory (see (8), (5)).

(iii) UV subtractions are independent of the dimensional parameters m and v, respectively.

(iv) SS and Ward identities for the spontaneously broken in LTP $U(N) \times U(n)_{\text{gauge}}$ symmetry are preserved by the 'soft mass' $1/N$ renormalization scheme.

We arrive at the following renormalized quantum generating functional ($\mathscr{R}$ denotes the graph by graph SS R-operation):

$$Z[J, L, K] = \mathscr{R}\{ \int \prod_{x,\theta} d\Phi\, d\Phi^*\, d\Sigma_0\, d\underset{\sim}{\Sigma}\, d\mathscr{A}_0\, d\underset{\sim}{\mathscr{A}}\, d\mathscr{B}_0\, d\underset{\sim}{\mathscr{B}}\, d\underset{\sim}{\chi}\, d\underset{\sim}{\chi}^* \times$$

$$\times \exp i \int d^3x\, d^2\theta\, [\mathfrak{L}'_{\text{ren}}(x, \theta) + J^*\Phi + \Phi^* J + (1 + \sigma_0) L_0 \Sigma_0 + (1 + \sigma_1) L_A \Sigma_A +$$

$$+ \bar{K}_0 \mathscr{A}_0 + \bar{K}_A \mathscr{A}_A + m\omega' L_0 + \omega_0 (2g_0)^{-1} L_0^2 + \omega_1 (2g_1)^{-1} L_A^2] \}; \tag{9}$$

$$\mathcal{L}'_{\text{ren}}(x,\theta) = \tfrac{1}{2}(1+b)(\overline{\nabla}^{(z)}_\alpha \Phi'^*)(\nabla^{(z)}_\alpha \Phi') - sm\Phi'^*\Phi' - \Phi'^*(\Sigma_0 + \underset{\sim}{\Sigma})\Phi' +$$

$$+ Nn(\mu/T + m/g_0 + ma)\Sigma_0 + Nn(1+c_0)(2g_0)^{-1}\Sigma_0^2 + N(1+c_1)(2g_1)^{-1}\,\mathrm{tr}\,(\underset{\sim}{\Sigma}^2) +$$

$$+ iNn(8e_0^2\mu)^{-1}\ \mathrm{tr}\,(\hat{\mathcal{F}}_0 \hat{\partial} \hat{\mathcal{F}}_0) + iN(8e_1^2\mu)^{-1}\ \mathrm{tr}\,(\underset{\sim}{\hat{\mathcal{F}}}^{(z)} \hat{\partial} \underset{\sim}{\hat{\mathcal{F}}}^{(z)} +$$

$$+ \tfrac{2}{3}(1+z)\underset{\sim}{\hat{\mathcal{F}}}^{(z)} \underset{\sim}{\hat{\mathcal{F}}}^{(z)} \underset{\sim}{\hat{\mathcal{F}}}^{(z)}) - Nn\mathcal{B}_0 \overline{\mathcal{D}}_\alpha \mathcal{A}_{0\alpha} - N\,\mathrm{tr}\,(\underset{\sim}{\mathcal{B}} \overline{\mathcal{D}}_\alpha \underset{\sim}{\mathcal{A}}_\alpha) +$$

$$+ \mathrm{tr}\,(\underset{\sim}{\chi}^* \overline{\mathcal{D}}_\alpha \nabla^{(z)}_\alpha \underset{\sim}{\chi}),$$

where

$$\Phi'^k_a \equiv \Phi^k_a + N^{1/2} s^{1/2} v^k_a, \qquad \nabla^{(z)}_\alpha \Phi \equiv \nabla_\alpha \Phi + iz\underset{\sim}{\mathcal{A}}_\alpha \Phi,$$

$$\nabla^{(z)}_\alpha \underset{\sim}{\chi} \equiv \nabla_\alpha \underset{\sim}{\chi} + iz[\underset{\sim}{\mathcal{A}}_\alpha, \underset{\sim}{\chi}], \ \underset{\sim}{\hat{\mathcal{F}}}^{(z)}_{\alpha\beta} \equiv \underset{\sim}{\hat{\mathcal{F}}}_{\alpha\beta} + iz\{\underset{\sim}{\mathcal{A}}_\alpha, \underset{\sim}{\overline{\mathcal{A}}}_\beta\}.$$

$a, b, c_{0,1}\ \sigma_{0,1}, \omega', \omega_{0,1}, z$ denote finite counterterm coefficients accounting for the subtraction ambiguity and they must be independent of the dimensional parameters if we are to retain property (iii) above. In particular, all of them can be set as equal to zero. Let us stress on the absence of gauge coupling constants renormalization (the latter have a positive mass dimension in $D = 2 + 1$).

5. From Equation (9) by means of a SS extension of the standard method of differential-vertex operations [11], one can easily derive the RG equations of SS HM_3:

$$\{\mu\partial/\partial\mu + \sum_{r=0,1} (h_r\partial/\partial h_r + \beta_r(\mathbf{u},\mathbf{h})\partial/\partial u_r) + \zeta_{\Sigma_0}(\mathbf{u},\mathbf{h})(m\partial/\partial m + L_0\delta/\delta L_0) +$$

$$+ \zeta_\Phi(\mathbf{u},\mathbf{h})(J\delta/\delta J + J^*\delta/\delta J^* - v\partial/\partial v - v^*\partial/\partial v^*) + \zeta_{\underset{\sim}{\Sigma}}(\mathbf{u},\mathbf{h})L_A\delta/\delta L_A\} \times \qquad (10)$$

$$\times\ W[J, L, K] = mu_0\rho'(\mathbf{u},\mathbf{h})L_0 + u_0\rho_0(\mathbf{u},\mathbf{h})L_0^2 + u_1\rho_1(\mathbf{u},\mathbf{h})L_A^2,$$

where

$$iW[J, L, K] = \log Z[J, L, K]; \qquad u_r \equiv 8/g_r, \qquad h_r \equiv 8/e_r^2, \quad r = 0, 1;$$

$$(\mathbf{u},\mathbf{h}) \equiv (u_0, u_1, h_0, h_1); \qquad \beta_r = 1/N\beta_r^{(1)} + \mathcal{O}(1/N^2), \qquad \zeta_{(\cdot)} = 1/N\zeta_{(\cdot)}^{(1)} + \mathcal{O}(1/N^2).$$

Now, to determine $\beta_r^{(1)}, \zeta_{(\cdot)}^{(1)}$ it is sufficient to use Equations (10) to the leading $1/N$ order for the particular cases of

$$\langle \Phi\Phi^*\Sigma_{0,A}\rangle = \delta^3 W[J, L, K]/\delta J^*\,\delta J\,\delta L_{0,A}\,|_{J=L=K=0}.$$

The results are as follows:

$$\zeta_{\Phi}^{(1)}(\mathbf{u},\mathbf{h}) = 1/n\, \zeta_{\Phi}^{(1)}(u_0, h_0) + (n^2-1)n^{-1}\zeta_{\Phi}^{(1)}(u_1, h_1),$$

$$\beta_0^{(1)}(\mathbf{u},\mathbf{h}) = 1/n\, \beta^{(1)}(u_0, h_0) + (n^2-1)n^{-1}\left[\frac{16u_1(1-3u_0^2-3u_0u_1)}{3\pi^2(1+u_1^2)^2} - 4u_0\zeta_{\Phi}^{(1)}(u_1,h_1)\right],$$

$$\beta_1^{(1)}(\mathbf{u},\mathbf{h}) = (n^2-2-c(n))n^{-1}\beta^{(1)}(u_1,h_1) + 1/n\{-4u_1[\zeta_{\Phi}^{(1)}(u_0,h_0) + (1+c(n))\zeta_{\Phi}^{(1)}(u_1,h_1)] +$$
$$+\, 8u_1(c(n)-2)[\pi^2(1+u_1^2)]^{-1} + 8[2u_0 + 8u_1 + 3u_1u_0^2 - 12u_0u_1^2 - 9u_1^3][3\pi^2(1+u_0^2)(1+u_1^2)]^{-1}\},$$

$$\zeta_{\Sigma_0}^{(1)}(\mathbf{u},\mathbf{h}) = 1/n\, \zeta_{\Sigma}^{(1)}(u_0, u_0) + (n^2-1)n^{-1}\left[-2\zeta_{\Phi}^{(1)}(u_1,h_1) - \frac{8u_1(u_1+2u_0)}{\pi^2(1+u_1^2)^2}\right], \tag{11}$$

$$\zeta_{\underset{\sim}{\Sigma}}^{(1)}(\mathbf{u},\mathbf{h}) = (n^2-2-c(n))n^{-1}\zeta_{\Sigma}^{(1)}(u_1,h_1) + 1/n\{-2[\zeta_{\Phi}^{(1)}(u_0,h_0) + (1+c(n))\zeta_{\Phi}^{(1)}(u_1,h_1)] +$$
$$+\,4(c(n)-2)[\pi^2(1+u_1^2)]^{-1} + 8[1+\tfrac{1}{2}(u_0^2 - 3u_1^2 + u_0^2u_1^2) - 3(u_0u_1 + u_0u_1^3 + u_1^4)][\pi^2(1+u_0^2)(1+u_1^2)]^{-1},$$

$$c(n)\delta_{AB} = f_{ACD}f_{BCD},$$

where

$$\zeta_{\Phi}^{(1)}(u,h) \equiv \frac{2}{\pi^2(1+u^2)} - \frac{4}{\pi^2(1+h^2)^2}[1+h^2 - \pi/2\, h(1-h^2) - 2h^2\log h],$$

$$\beta^{(1)}(u,h) \equiv -\,4u\zeta_{\Phi}^{(1)}(u,h) + 16u(1-6u^2)[3\pi^2(1+u^2)^2]^{-1}, \tag{12}$$

$$\zeta_{\Sigma}^{(1)}(u,h) \equiv -\,2\zeta_{\Phi}^{(1)}(u,h) - 24u^2[\pi^2(1+u^2)^2]^{-1}.$$

From Equations (11) and (12) we find that there are only two stable fixed points of RG★:

$$\text{an IR one:}\quad u_r^* = 0, \qquad h_r^* = 0 \quad (\text{i.e., } g_r^* = \infty,\ e_r^* = \infty), \text{ cf., (7);}$$
$$\text{a UV one:}\quad u_r^{**} = \infty, \qquad h_r^{**} = \infty \quad (\text{i.e., } g_r^{**} = 0,\ e_r^{**} = 0 - \text{'asymptotic freedom'}). \tag{13}$$

For the two independent critical exponents $\eta = 2\zeta_{\Phi}(\mathbf{0},\mathbf{0})$, $\nu = 1 - \zeta_{\Sigma_0}(\mathbf{0},\mathbf{0}))^{-1}$ [6] we get in the leading $1/N$ order: $\eta^{(1)} = -4n(N\pi^2)^{-1}$, $\nu^{(1)} = 1 + 4n(N\pi^2)^{-1}$.

★Let us note that in the case of SS U(N) $(\Phi_a^*\Phi_a)^2_{D=2+1}$ model

$$\mathcal{L}(x,\theta) = \tfrac{1}{2}\bar{\mathcal{D}}_\alpha\Phi_a^*\mathcal{D}_\alpha\Phi_a - g/2N(\Phi_a^*\Phi_a - N\mu/T)^2$$

one gets Equations (12) with $\zeta_{\Phi}^{(1)} \equiv \zeta_{\Phi}^{(1)}(u) = 2[\pi^2(1+u^2)]^{-1}$. Thus unlike SS HM$_3$ $u^* = 0$ ($g^* = \infty$) becomes a UV stable fixed point (the corresponding limiting theory being the known SS U(N) $D = 2+1$ non-linear sigma model, cf. [12]) whereas $u^{**} = \infty$ ($g^{**} = 0$) is an IR one.

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ERRATA to 'Non-Perturbative Renormalization Group Functions in (2 + 1)-Dimensional Supersymmetric Gauge Theories' by E.R. Nissimov and S.J. Pacheva, *LMP* **6** (1982), 101–108.

There is a missing term in the right-hand side of Equation (1′):

$$-g_1(g_1 + 2g_0)(Nn)^{-2}(\varphi^*\varphi - Nn\mu/T)(\varphi^*\lambda_A\varphi)^2 .$$

In Equations (11), the following terms should be added to the expressions for:

$$\beta_0^{(1)}(\mathbf{u}, \mathbf{h}): \quad -(n^2 - 1)(n\pi^2)^{-1} 16u_0 \partial/\partial h_1 \, [h_1 I(h_1)] ,$$

$$\beta_1^{(1)}(\mathbf{u}, \mathbf{h}): \quad -[n\pi^2(h_0 - h_1)]^{-1} 16u_1 [h_0 I(h_0) - h_1 I(h_1)] ,$$

$$\zeta_{\Sigma_0}^{(1)}(\mathbf{u}, \mathbf{h}): \quad -(n^2 - 1)(n\pi^2)^{-1} 8\partial/\partial h_1 \, [h_1 I(h_1)] ,$$

$$\zeta_{\underset{\approx}{\Sigma}}^{(1)}(\mathbf{u}, \mathbf{h}): \quad -[n\pi^2(h_0 - h_1)]^{-1} 8[h_0 I(h_0) - h_1 I(h_1)] ,$$

where $I(h) \equiv (1 + h^2)^{-2}[1 + h^2 - \frac{1}{2}\pi h(1 - h^2) - 2h^2 \log h]$.

In Equations (12), the following terms should be added to the expressions for:

$$\beta^{(1)}(u, h): \quad -16u\pi^{-2}\partial/\partial h \, [hI(h)] ,$$

$$\zeta_{\Sigma}^{(1)}(u, h): \quad -8\pi^{-2}\partial/\partial h \, [hI(h)] .$$

Equation (13) should read accordingly:

an UV one: $u_r^* = 0, \quad h_r^* = \infty$ (i.e., $g_r^* = \infty$, $e_r^* = 0$);

an IR one: $u_r^{**} = \infty, \quad h_r^{**} = 0$ (i.e., $g_r^{**} = 0$, $e_r^{**} = \infty$).

Finally, the value of $\nu^{(1)}$ should be altered to: $\nu^{(1)} = 1 - 4n(N\pi^2)^{-1}$.

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